

Position & Source: Position vector \vec{r} , source vector \vec{r}' , separation vector $\vec{\Delta r} = \vec{r} - \vec{r}'$

Fundamental Theorems of Vector Calculus:

$$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \vec{dl} = f(\vec{b}) - f(\vec{a}) \quad \int \nabla \cdot \vec{A} \, d\tau = \oint \vec{A} \cdot \vec{da} \quad \int (\nabla \times \vec{A}) \cdot \vec{da} = \oint \vec{A} \cdot \vec{dl}$$

Cartesian Coordinates: $\vec{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z}$ $d\tau = dx \, dy \, dz$

$$\nabla f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z} \quad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{z}$$

Spherical Coordinates: $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$

$$\vec{dl} = dr\hat{r} + r \, d\theta\hat{\theta} + r \sin\theta \, d\phi\hat{\phi} \quad d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin\theta} \frac{\partial f}{\partial \phi} \hat{\phi} \quad \nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial(\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r \sin\theta} \left(\frac{\partial(\sin\theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right) \hat{r} + \frac{1}{r} \left(\frac{1}{\sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial(r A_\phi)}{\partial r} \right) \hat{\theta} + \frac{1}{r} \left(\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \hat{\phi}$$

Cylindrical Coordinates: $x = s \cos\phi$, $y = s \sin\phi$, $z = z$

$$\vec{dl} = ds\hat{s} + s \, d\phi\hat{\phi} + dz\hat{z} \quad d\tau = s \, ds \, d\phi \, dz$$

$$\nabla f = \frac{\partial f}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z} \quad \nabla \cdot \vec{A} = \frac{1}{s} \frac{\partial(s A_s)}{\partial s} + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{1}{s} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right) \hat{\phi} + \frac{1}{s} \left(\frac{\partial(s A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi} \right) \hat{z}$$

Dirac Delta Function: $\delta^3(\vec{\Delta r}) = \frac{1}{4\pi} \nabla \cdot \left(\frac{\vec{\Delta r}}{\Delta r^3} \right)$, $\int \delta^3(\vec{\Delta r}) \, d\tau = 1$ if \vec{r}' contained in volume

Irrotational Functions: $\nabla \times \vec{A} = 0$ $\vec{A} = \nabla f$ $\oint \vec{A} \cdot \vec{dl} = 0$

Solenoidal Functions: $\nabla \cdot \vec{F} = 0$ $\vec{F} = \nabla \times \vec{A}$ $\oint \vec{F} \cdot \vec{da} = 0$

Electric Field: $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{\Delta r^2} \widehat{\Delta r} d\tau'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}')}{\Delta r^2} \widehat{\Delta r} da'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}')}{\Delta r^2} \widehat{\Delta r} dl'$

Gauss's Law: $\oint \vec{E} \cdot d\vec{a} = Q_{enc}/\epsilon_0$, $\nabla \cdot \vec{E} = \rho/\epsilon_0$

Electric Potential: $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{\Delta r} d\tau'$, $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}')}{\Delta r} da'$, $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}')}{\Delta r} dl'$
 $\vec{E} = -\nabla V$, $V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$, $\nabla^2 V = -\rho/\epsilon_0$

Boundary Conditions: $\Delta \vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$, $\Delta V = 0$

Work and Energy: $W = Q\Delta V$, $W = \frac{\epsilon_0}{2} \int E^2 d\tau$